On Resonance of Dual-Spin Stabilized Projectiles Equipped with Canards

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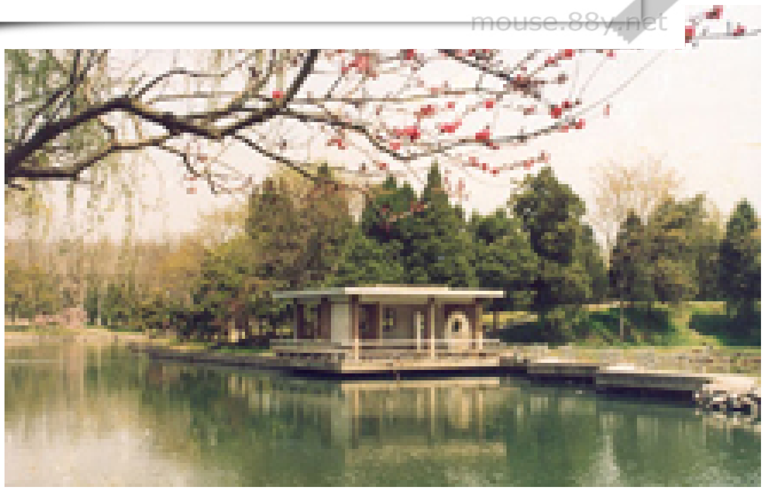
Beautiful Scenery of Our Campus

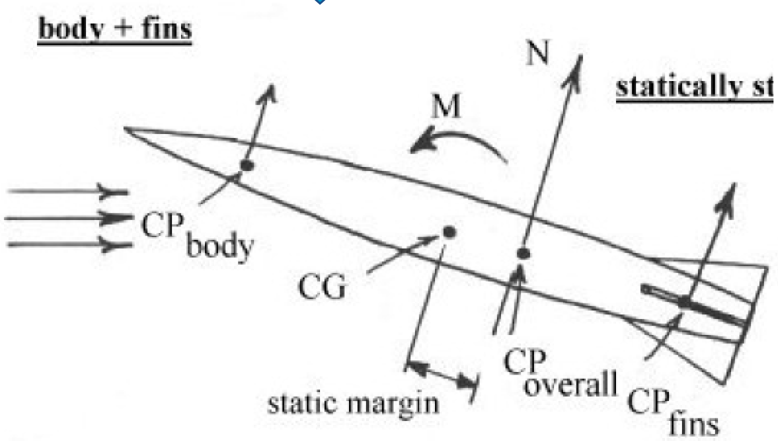


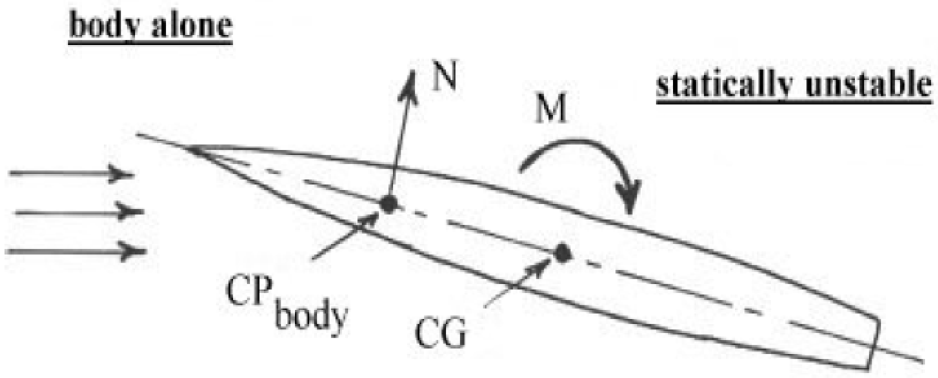
OUTLINE

INTRODUCTION

- Motivation: to improve the strike precision of conventional spin-stabilized projectiles
- Difficult Point: the inherent ballistic characteristics of spin-stabilized projectiles

• fin-stabilized projectile

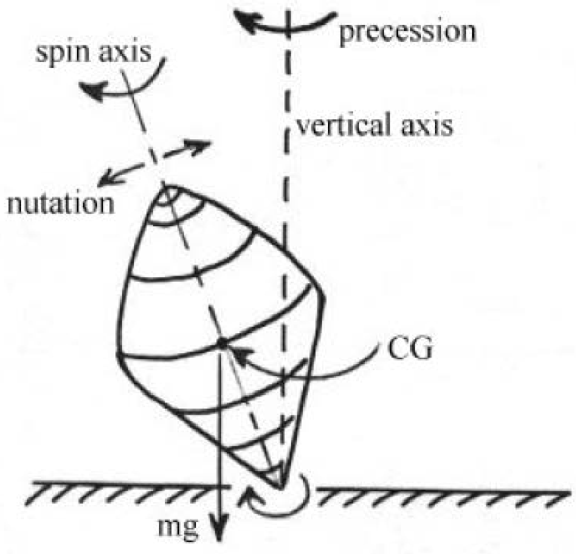




• spin-stabilized projectile

INTRODUCTION

- **How to keep the flight stability for spin-stabilized projectiles**

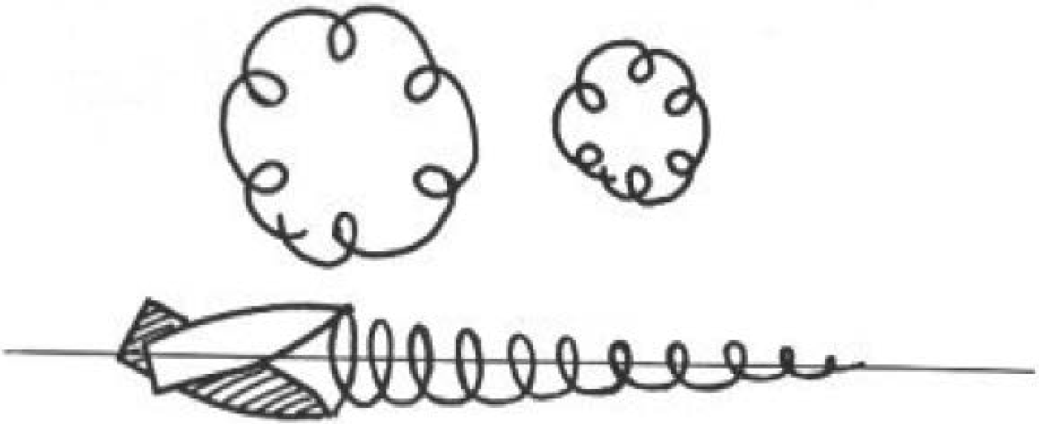


Gyroscopic Stabilization on a Spinning Top

Gyroscopic effect

Very high spin rate

(up to 10000 round per minute)



Nutation of a projectile subjected to an initial disturbance

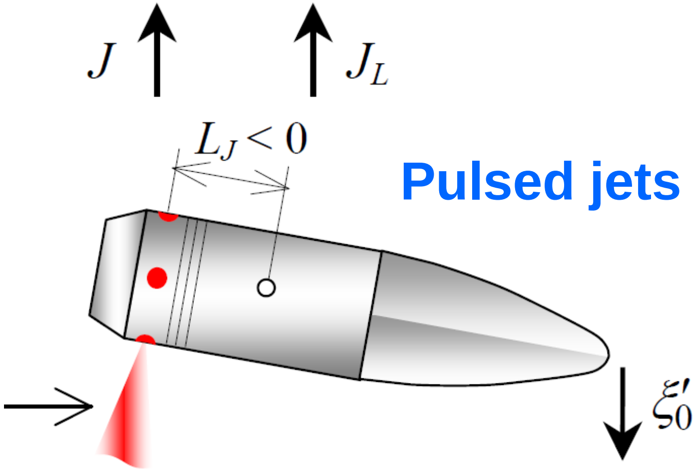
INTRODUCTION

- Some control mechanisms for spin-stabilized projectiles

C.Grignon,etal.

IMPROVEMENT OF ARTILLERY PROJECTILE ACCURACY

Drag brake



Pulsed jets

Pierre Wey, etal.

TRAJECTORY DEFLECTION OF FIN- AND SPIN-STABILIZED PROJECTILES USING PAIRED LATERAL IMPULSES

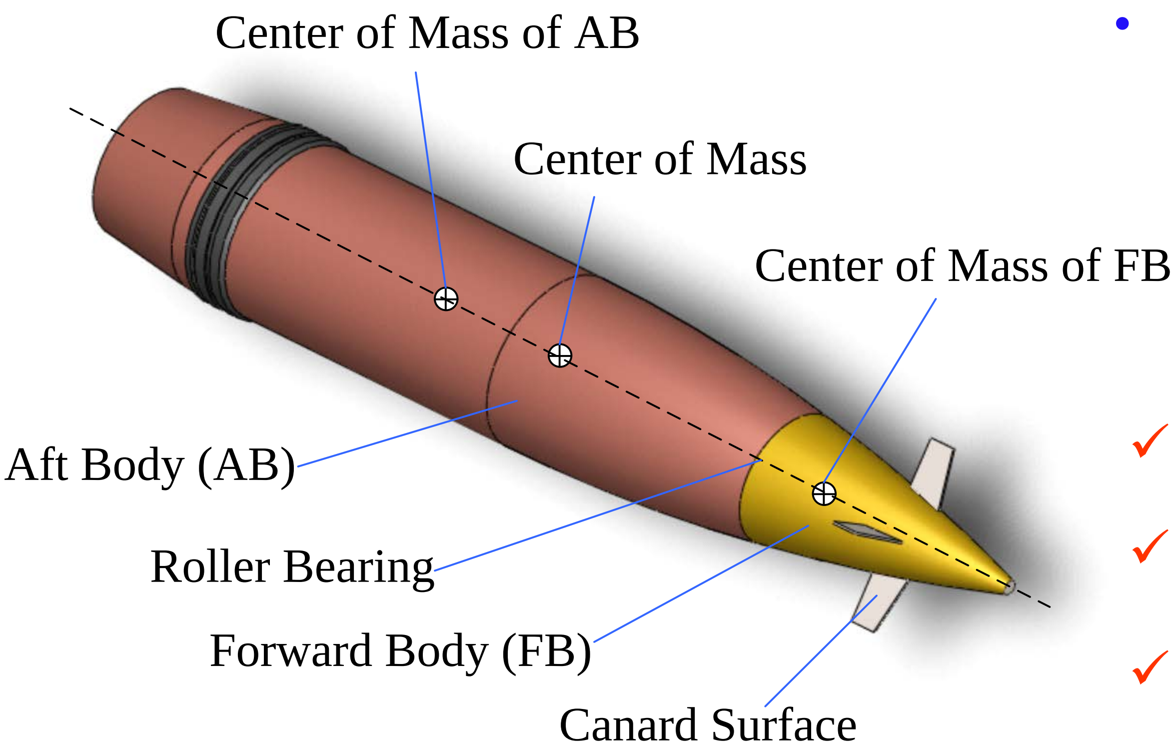
Thomas Pettersson, etal.

AERODYNAMICS AND FLIGHT STABILITY FOR A COURSE CORRECTED PROJECTILE ROUND

Spin brakes

INTRODUCTION

- **A dual-spin-stabilized projectile**



- **It can change the trajectory accurately and continuously**

✓ **Roll attitude measurement**

✓ **Application of canards**

✓ **Flight stability**

CURRENT LITERATURE

Wernert, P., Leopold F., and Bidino, D., etc. 2008. "Wind Tunnel Tests and Open-Loop Trajectory Simulations for a 155 mm Canards Guided Spin Stabilized Projectile," AIAA Atmospheric Flight Mechanics Conference and Exhibit, AIAA Paper 2008-6881, Honolulu, Hawaii.

Wernert, P. 2009. "Stability Analysis for Canard Guided Dual-Spin Stabilized Projectiles," AIAA Atmospheric Flight Mechanics Conference, AIAA Paper 2009-5843, Chicago, Illinois.

Wernert, P., and Theodoulis, S. 2011. "Modeling and Stability Analysis for a Class of 155 mm Spin-Stabilized Projectiles with Course Correction Fuse (CCF)," AIAA Atmospheric Flight Mechanics Conference, AIAA Paper 2011-6269, Portland, Oregon.

Theodoulis, S., and Wernert, P. 2011. "Flight Control for a Class of 155 mm Spin-Stabilized Projectiles with Course Correction Fuse (CCF)," AIAA Guidance, Navigation and Control Conference, AIAA Paper 2011-6247, Portland, Oregon.

OUR RECENT WORK (2014-2016)

Chang, S., Qian, L., and Wang, Z. 2014. “Modeling and Simulation of Ballistic Characteristics for Dual-Spin Stabilized Projectiles Equipped with Canards,” Proceedings of the 28th International Symposium on Ballistics. Atlanta, USA: IBS, pp. 557-567.

Chang, S., Wang, Z., and Liu, T. 2014. “Analysis of Spin-Rate Property for Dual-Spin-Stabilized Projectiles with Canards,” Journal of Spacecraft and Rockets, Vol. 51, No. 3, pp. 958-966.

Chang, S. 2016. “Dynamic Response to Canard Control and Gravity for a Dual-Spin Projectile”, Journal of Spacecraft and Rockets, Accepted.

Chang, S., Wang, Z., and Liu, T. 2016. “A Theoretical Study on Forced Motion for Dual-Spin-Stabilized Projectiles with Canards”, ACTA ARMAMENTARII, Accepted.

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PROJECTILE AND CANARD DYNAMIC MODEL

- **Assumptions and Simplification**
- The forward and aft bodies of dual-spin stabilized projectiles are described separately
- the principle axes of inertia of the forward and aft bodies are parallel to those of the combination
- the coupled effect of aerodynamics acted on projectile body is not considered, either.

PROJECTILE AND CANARD DYNAMIC MODEL

- **Projectile Dynamic Model**
- In terms of Newton's second law
- With respect to the no-roll reference frame (NRRF)

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{F_\xi}{m} + g_\xi + (rv - qw) \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{F_\eta}{m} + g_\eta - ru \\ \dfrac{\mathrm{d}w}{\mathrm{d}t} = \dfrac{F_\zeta}{m} + g_\zeta + qu \end{cases}$$

PROJECTILE AND CANARD DYNAMIC MODEL

- **Projectile Dynamic Model**
- With respect to the no-roll reference frame (NRRF)

$$\begin{cases} \dfrac{\mathrm{d}p_F}{\mathrm{d}t} = \dfrac{M_{F\xi} + M_V}{C_F}, \quad \dfrac{\mathrm{d}p_A}{\mathrm{d}t} = \dfrac{M_{A\xi} - M_V}{C_A} \\ \dfrac{\mathrm{d}q}{\mathrm{d}t} = \dfrac{M_\eta}{\tilde{A}}, \quad \dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{M_\zeta}{\tilde{A}} \end{cases}$$

M_V is the roll constrain moment

$$\begin{cases} \tilde{A} = A_F + m_F \cdot r_F{}^2 + A_A + m_A \cdot r_A{}^2 \\ M_\eta = M_{F\eta} + M_{A\eta} - r\left(p_F C_F + p_A C_A\right) \\ M_\zeta = M_{F\zeta} + M_{A\zeta} + q\left(p_F C_F + p_A C_A\right) \end{cases}$$

PROJECTILE AND CANARD DYNAMIC MODEL

- **Canard Dynamic Model**
- With respect to the no-roll reference frame (NRRF)

$$\delta_{C\xi} = 0$$

The canard deflection angles

$$\ldots s\,\gamma_F - \delta_{CZ1F} \cdot \sin\gamma_F$$

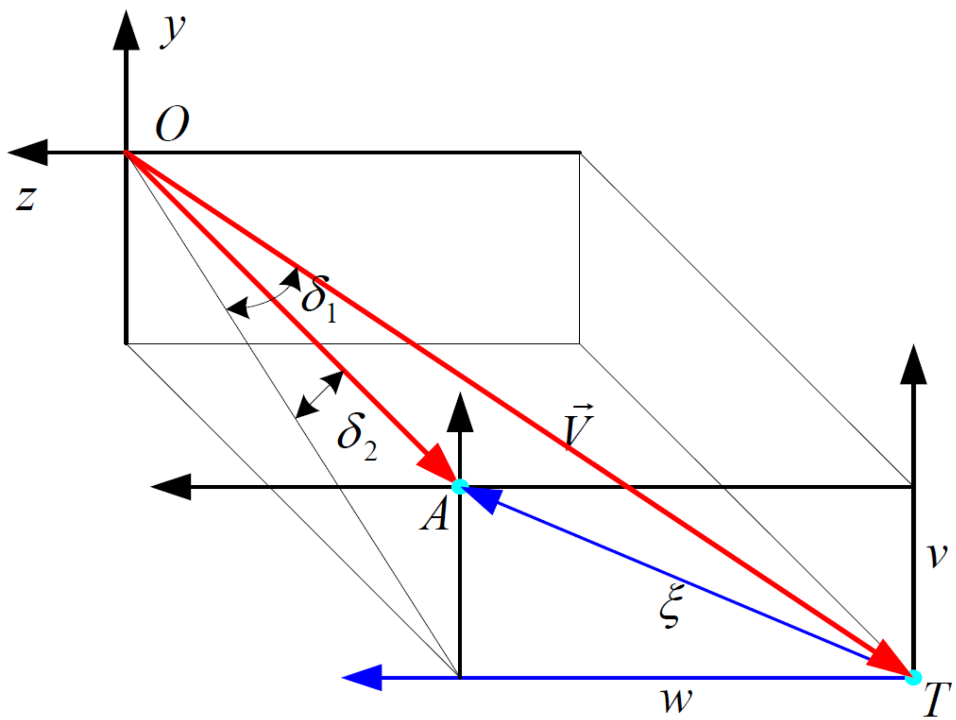
$$\ldots n\,\gamma_F + \delta_{CZ1F} \cdot \cos\gamma_F$$

$$\begin{cases} F_{C_2\xi} = QV^2 C_N^{\delta_c} \delta_{C\xi} \\ F_{C_2\eta} = QV^2 C_N^{\delta_c} \left(\delta_{C\eta} - v/u\right) \\ F_{C_2\zeta} = QV^2 C_N^{\delta_c} \left(\delta_{C\zeta} - w/u \ldots\right. \end{cases}$$

$$\begin{cases} M_{C_2\xi} = QV^2 C_N^{\delta_c} \cdot l_{PG} \cdot \delta_{C\xi} \\ M_{C_2\eta} = QV^2 C_N^{\delta_c} \cdot l_{PG} \left(-\delta_{C\zeta} + w/u\right) \\ M_{C_2\zeta} = QV^2 C_N^{\delta_c} \cdot l_{PG} \left(\delta_{C\eta} - v/u\right) \end{cases}$$

DYNAMIC MODEL OF ANGLE OF ATTACK

- The generalized angle of attack is used to be the independent variable



$$\xi = \left(-\frac{v}{V}\right) + i\left(-\frac{w}{V}\right)$$

- Geometric description of the generalized angle of attack for the dual-spin projectile

DYNAMIC MODEL OF ANGLE OF ATTACK

- Using the transverse differential equations and taking the trajectory arc as argument, we can obtain the transverse equations of dual-spin stabilized projectiles with complex form as follows:

$$\begin{cases} \dfrac{v'+i\cdot w'}{V}-i\dfrac{u}{V}\left(\dfrac{q+i\cdot r}{V}\right)=\dfrac{F_Y+i\cdot F_Z}{mV^2}+\dfrac{g_Y+ig_Z}{V^2} \\ \dfrac{q'+i\cdot r'}{V}-i\dfrac{\left(C_F p_F+C_A p_A\right)}{\tilde{A}V}\left(\dfrac{q+i\cdot r}{V}\right)=\dfrac{\left(M_{FY}+M_{AZ}\right)+i\left(M_{FZ}+M_{AZ}\right)}{\tilde{A}V^2} \end{cases}$$

DYNAMIC MODEL OF ANGLE OF ATTACK

$$\begin{cases} \xi' + \xi\left(\dfrac{V'}{V}\right) + i\eta\mu = -\dfrac{\left(F_Y + iF_Z\right)}{mV^2} - \dfrac{\left(g_Y + ig_Z\right)}{V^2} \\ \mu' + \mu\left(\dfrac{V'}{V}\right) - i\tilde{P}\mu = \dfrac{\left(M_{FY} + M_{AY}\right) + i\left(M_{FZ} + M_{AZ}\right)}{\tilde{A}V^2} \end{cases}$$

Deducing

$$\begin{cases} \mu = \dfrac{-1}{i\eta}\left[\xi' + \left(\eta b_y - \dfrac{g\sin\theta}{V^2}\right)\xi + \dfrac{g_\Delta}{V^2}\right] \\ \mu' = \dfrac{-1}{i\eta}\left\{\xi'' + \left(\eta b_y - \dfrac{g\sin\theta}{V^2}\right)\xi' + \left[\eta' b_y - \left(\dfrac{g\sin\theta}{V^2}\right)' + \eta b_y'\right]\xi + \left(\dfrac{g_\Delta}{V^2}\right)'\right\} \\ \qquad + \dfrac{\eta'}{i\eta^2}\left[\xi' + \left(\eta b_y - \dfrac{g\sin\theta}{V^2}\right)\xi + \dfrac{g_\Delta}{V^2}\right] \end{cases}$$

Deducing

……

Deducing

DYNAMIC MODEL OF ANGLE OF ATTACK

- the model of nonlinear angular motion for dual-spin-stabilized projectiles

$$\boxed{\xi'' + \left(H - \frac{\eta'}{\eta} - iP_D \right)\xi' - \left(M + iP_D \cdot T \right)\xi = R}$$

$$M = \eta\left(k_z - b_y'\right) + M_C \qquad H = \left(\eta b_y - \frac{g\sin\theta}{V^2}\right) + \frac{N_C}{\eta} - \left(b_x + \frac{g\sin\theta}{V^2} - k_{zz}\right)$$

$$T = \left(\eta b_y - \frac{g\sin\theta}{V^2} + \frac{N_C}{\eta}\right) - \frac{\eta}{P_D}\frac{A_A p_A}{\tilde{A} V} k_y$$

$$R = -\left(\frac{g_\eta + ig_\zeta}{V^2}\right)' + \left(\frac{g_\eta + ig_\zeta}{V^2} + N_C \delta_C e^{i\omega_F \cdot s}\right)\left(\frac{\eta'}{\eta} + iP_D + b_x + \frac{g\sin\theta}{V^2} - k_{zz}\right)$$

$$+ \eta M_C \delta_C e^{i\omega_F \cdot s} - iN_C \delta_C e^{i\omega_F s}\left(\frac{p_F}{V}\right)$$

ANALYSIS OF RESONANCE

- Simplification using projectile linear theory

$$\left(\frac{g_\eta + i g_\zeta}{V^2}\right)' \approx \frac{\ddot{\theta}}{V^2} - \frac{\dot{\theta}}{V}\left(-b_x - \frac{g\sin\theta}{V^2}\right),$$

$$g_\eta + i g_\zeta \approx -g\cos\theta, \quad \eta = 1, \quad \eta' = 0$$

$$\xi = \left(-\frac{v}{V}\right) + i\left(-\frac{w}{V}\right) \approx \delta_2 + i\delta_1$$

ANALYSIS OF RESONANCE

- A Linearized Model of the Pitching and Yawing Motion

$$\boxed{\xi'' + \left(H - iP_D\right)\xi' - \left(M + iP_D \cdot T\right)\xi = R}$$

$$\begin{cases} H = b_y - b_x + k_{zz} - 2\dfrac{g\sin\theta}{V^2} + N_C, \qquad P_D = \dfrac{C_F p_F + C_A p_A}{\tilde{A}V} \\ T = \left(b_y - \dfrac{g\sin\theta}{V^2} + N_C\right) - \dfrac{k_y}{P_D}\dfrac{A_A p_A}{\tilde{A}V}, \qquad M = k_z + M_C \\ R = -\dfrac{\ddot{\theta}}{V^2} + \dfrac{\dot{\theta}}{V}\left(-b_x - \dfrac{g\sin\theta}{V^2}\right) + \left(-\dfrac{g\cos\theta}{V^2}\right)\left(iP_D + b_x + \dfrac{g\sin\theta}{V^2} - k_{zz}\right) \\ \qquad + \left[M_C\delta_C + i\left(P_D - \dfrac{p_F}{V}\right)N_C\delta_C\right]e^{i\omega_F \cdot s} \end{cases}$$

ANALYSIS OF RESONANCE

- Analytical Solution of Periodical Action by Canards

Forced Term

$$R_P = \left[M_C \delta_C + i \left(P_D - \frac{p_F}{V} \right) N_C \delta_C \right] e^{i\omega_F \cdot s}$$

Analytical Solution

$$\xi_P = \frac{M_C \delta_C + i \left(P_D - \frac{p_F}{V} \right) N_C \delta_C}{-\omega_F^2 + i\left(H - iP_D\right)\omega_F - \left(M + iP_D T\right)} e^{i\omega_F s}$$

ANALYSIS OF RESONANCE

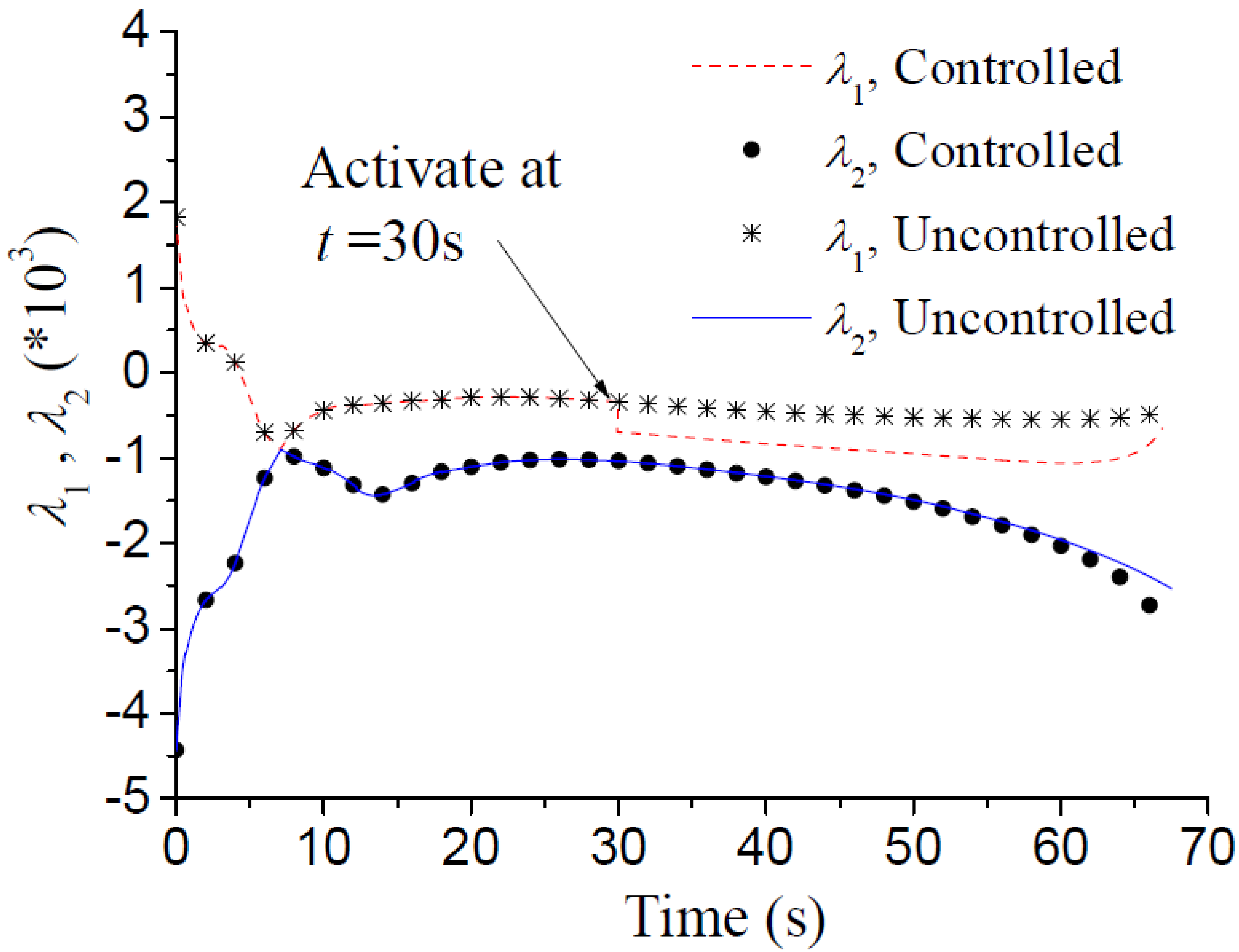
- Analytical Solution of Periodical Action by Canards

$$\xi_P = \frac{\left[M_C\delta_C + i\left(P_D - \frac{p_F}{V}\right)N_C\delta_C\right]e^{i\omega_F s}}{\left[i\left(\omega_F - \omega_1\right) - \lambda_1\right]\left[i\left(\omega_F - \omega_2\right) - \lambda_2\right]}$$

the amplitude of ξ_P

$$\left|\xi_P\right| = \frac{\left[M_C\delta_C + i\left(P_D - \frac{p_F}{V}\right)N_C\delta_C\right]}{\sqrt{\left[\left(\omega_F - \omega_1\right)^2 + \lambda_1{}^2\right]\left[\left(\omega_F - \omega_2\right)^2 + \lambda_2{}^2\right]}}$$

NUMERICAL SIMULATION



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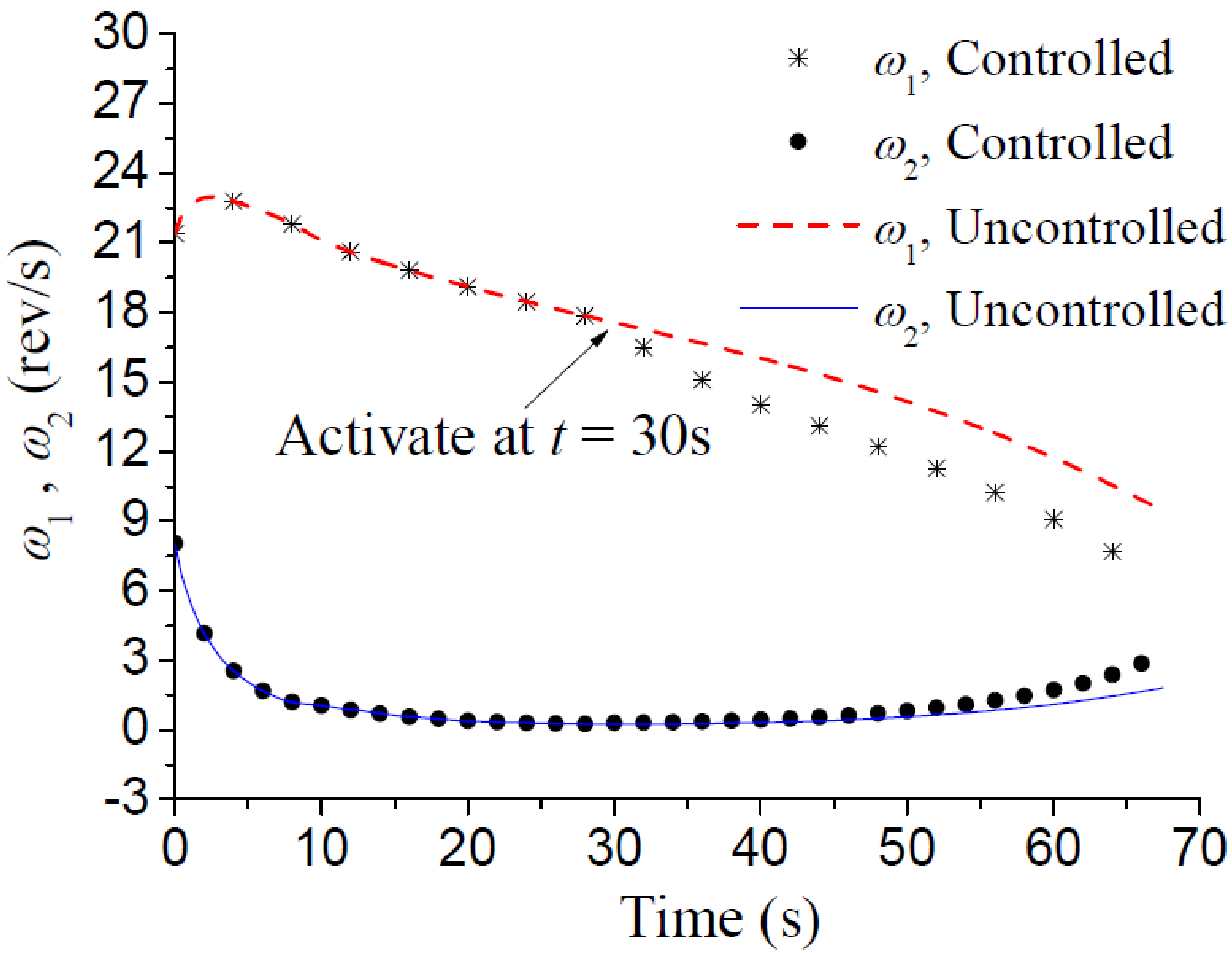

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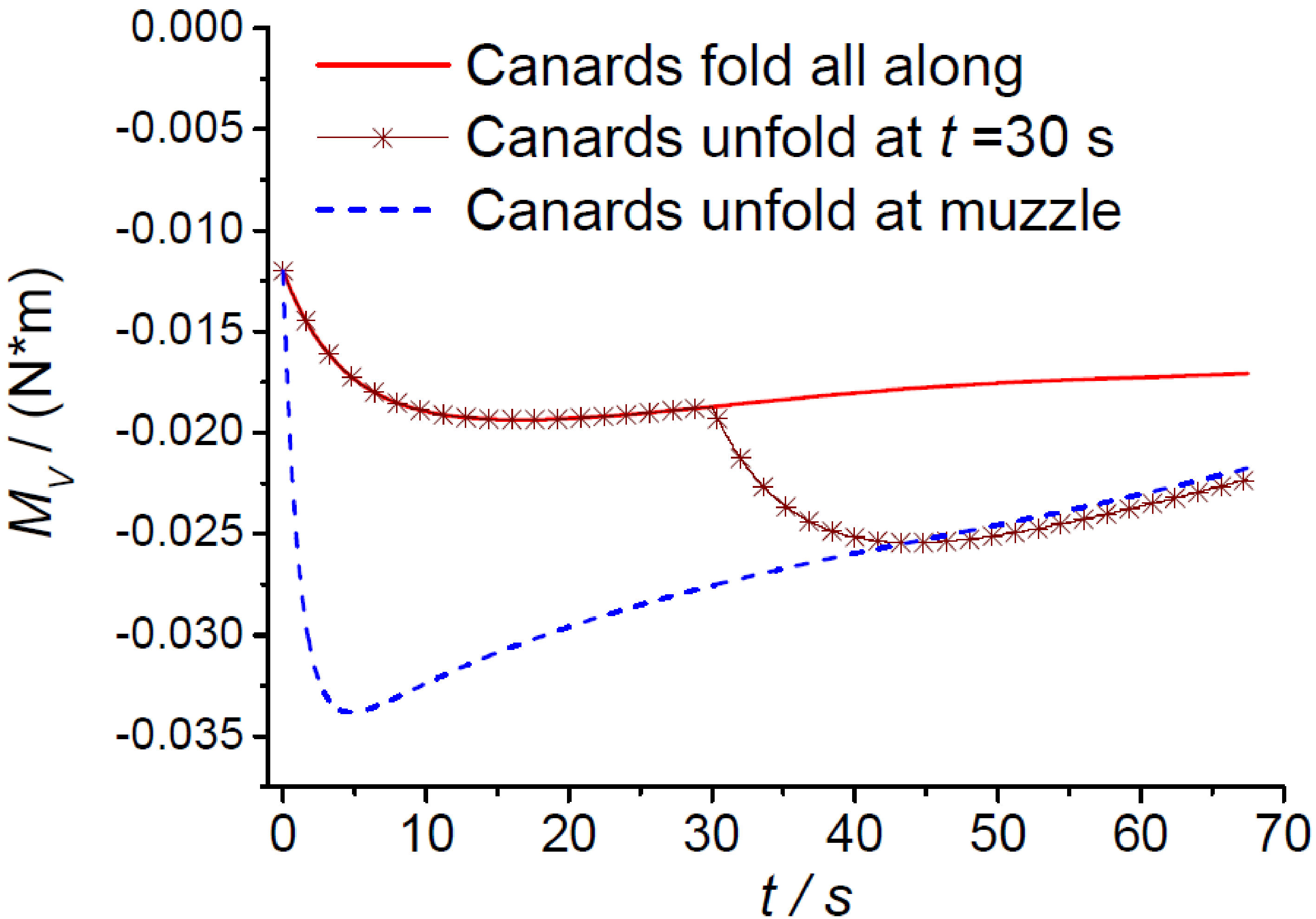

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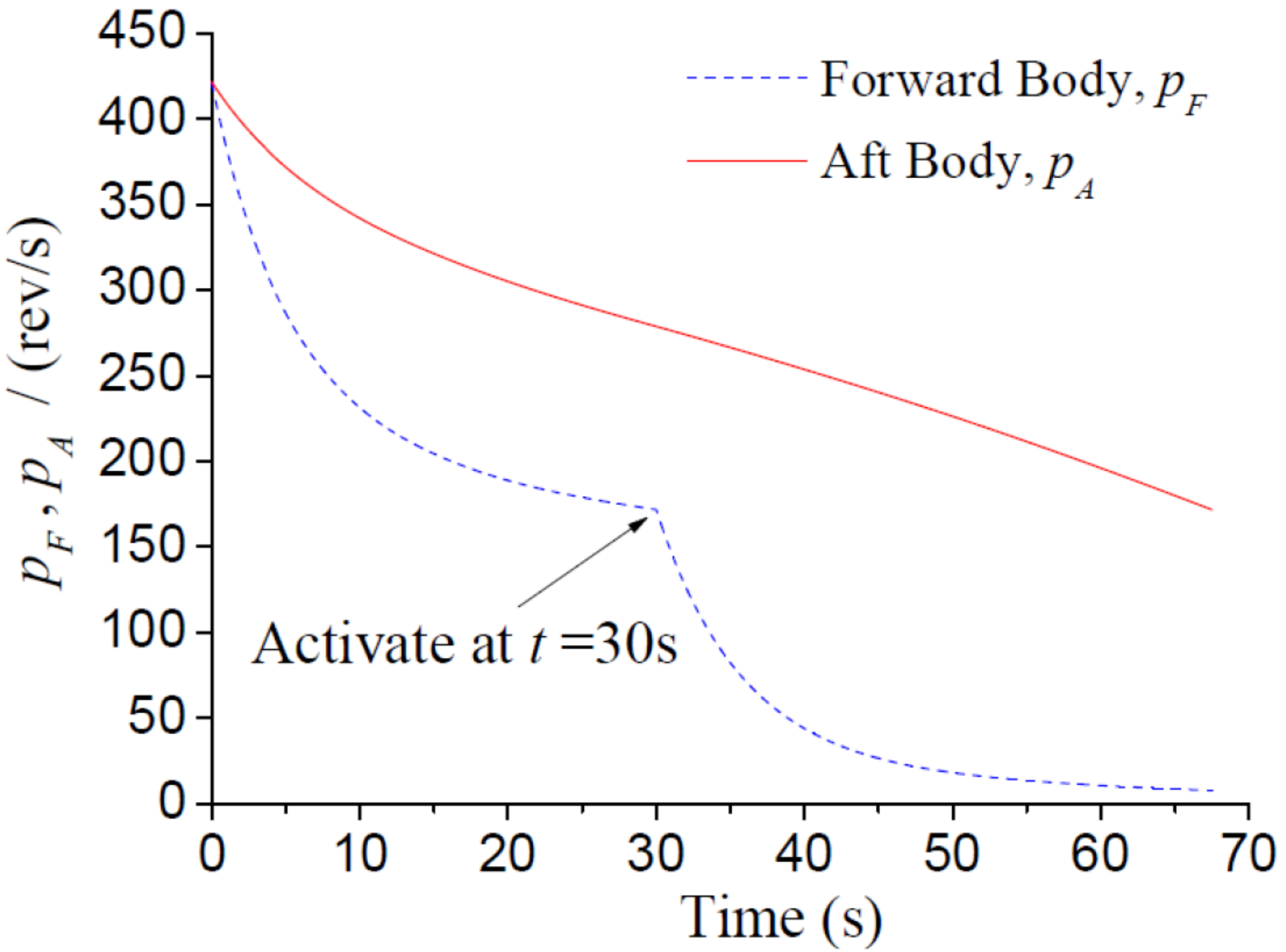

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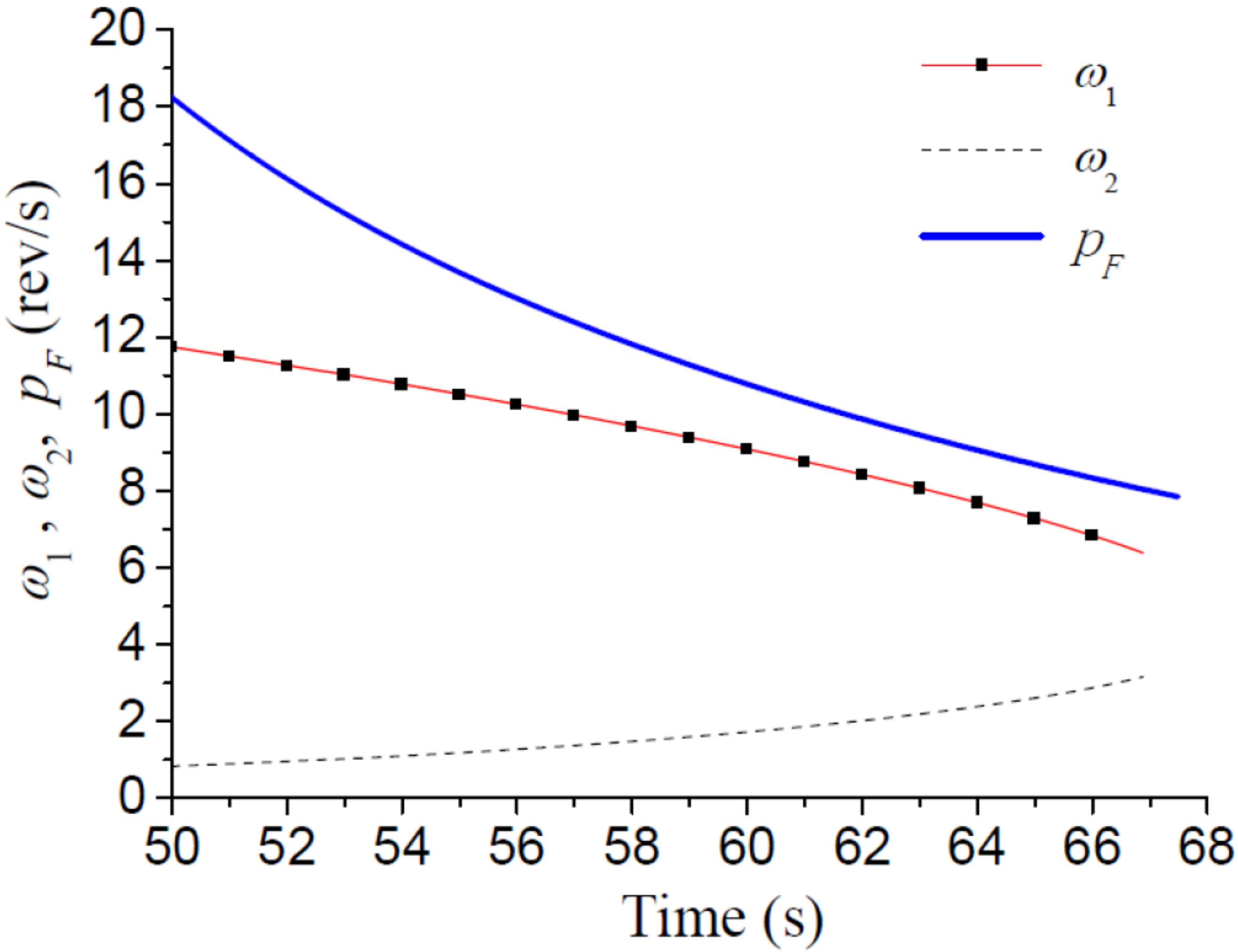

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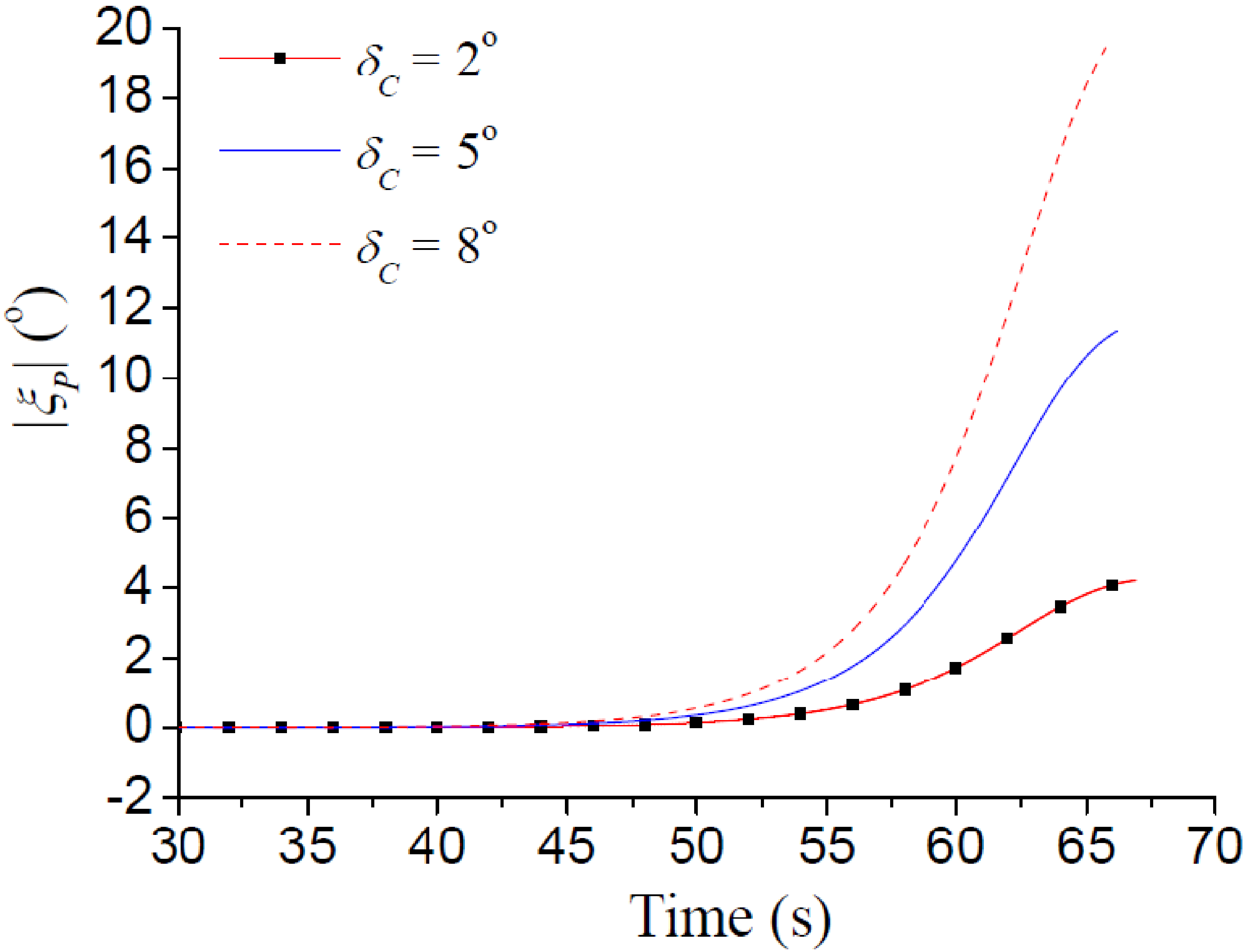

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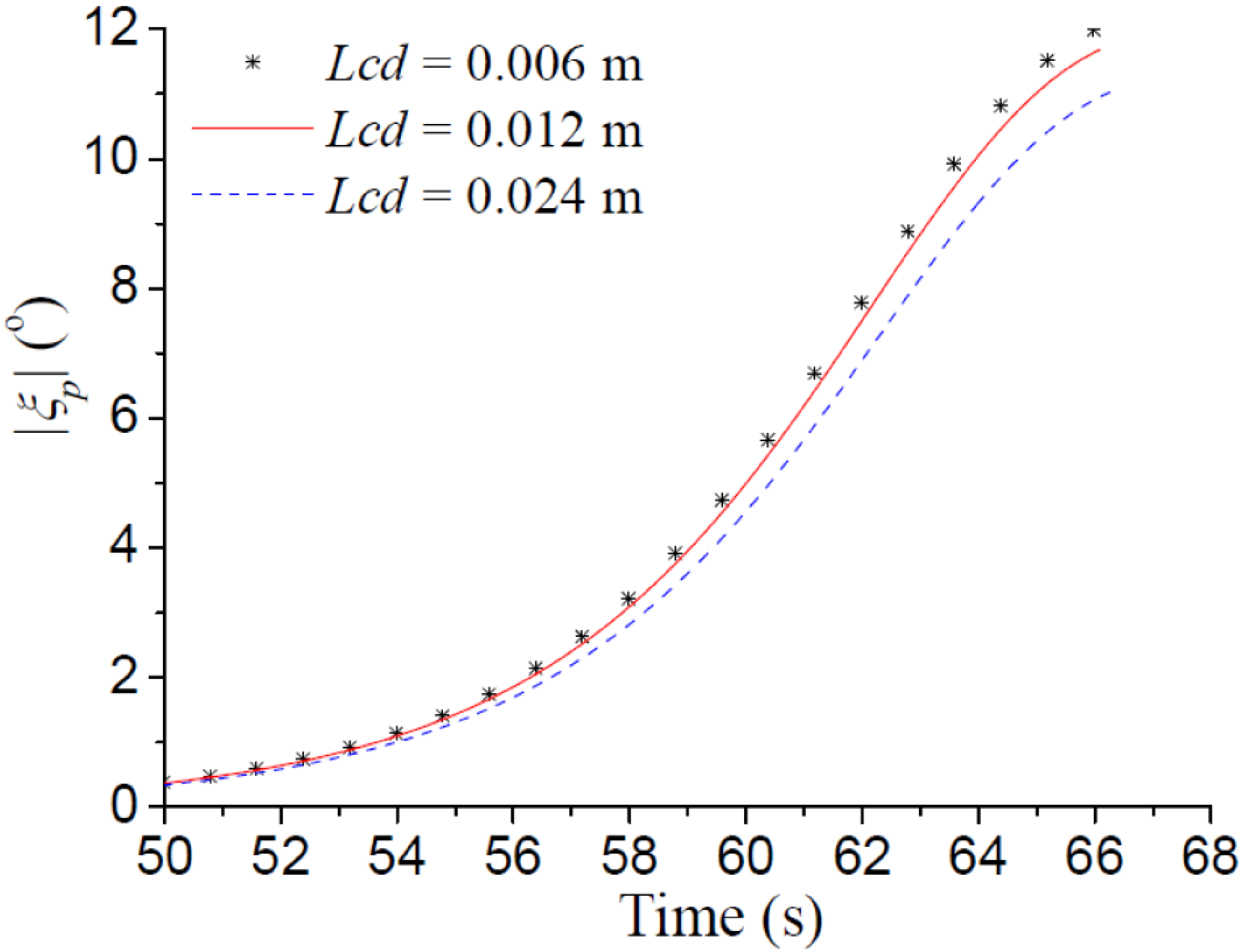

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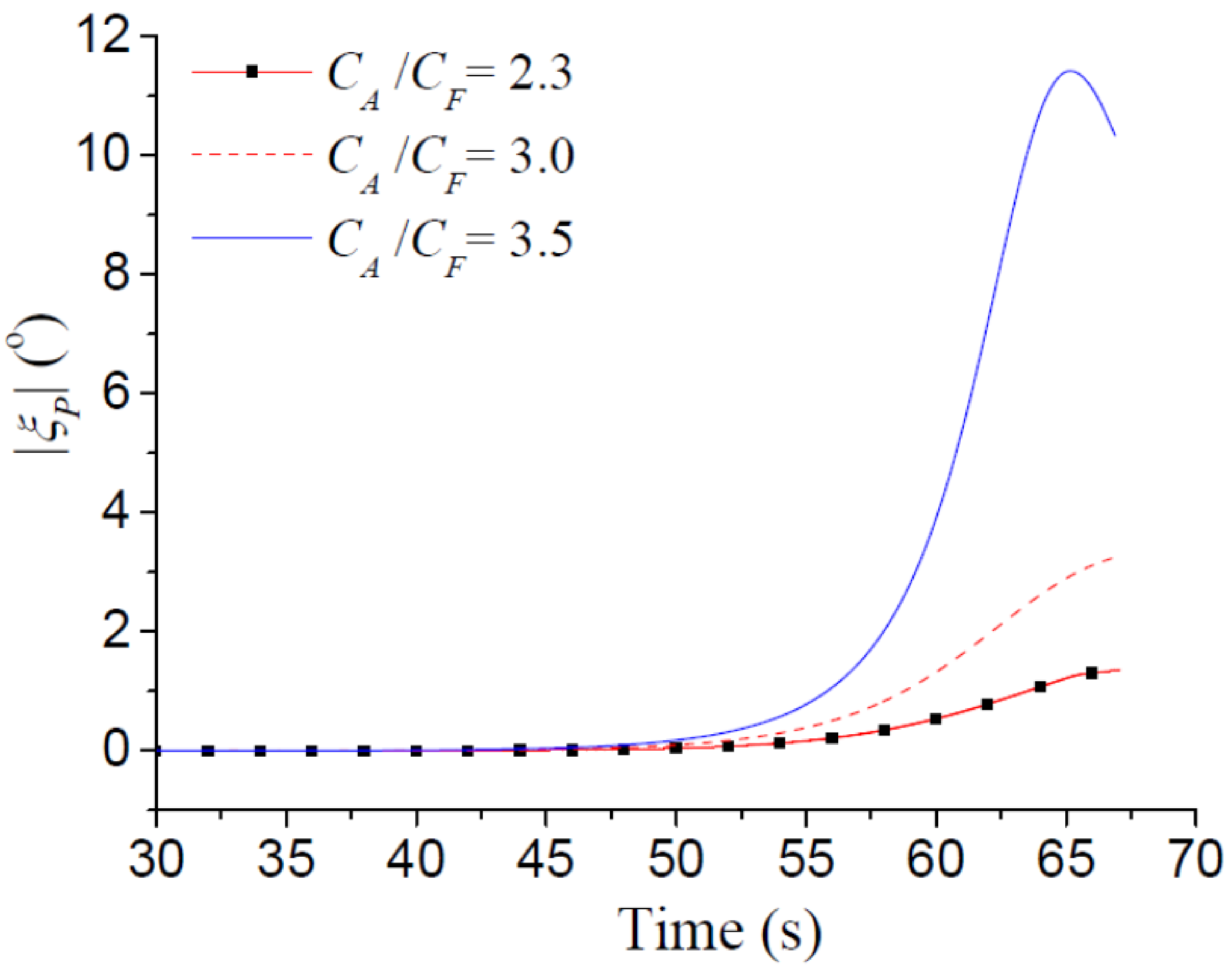

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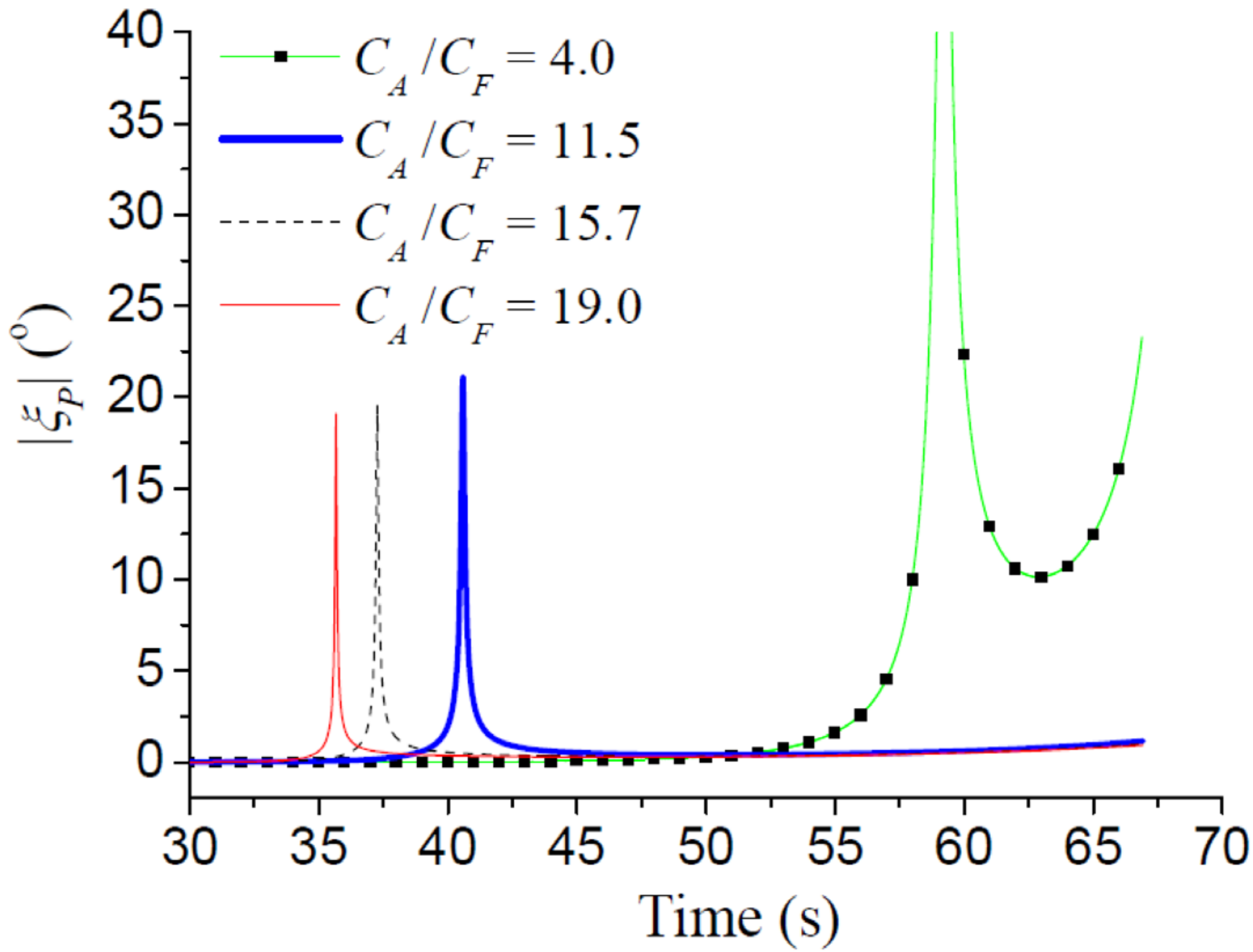

CONCLUSIONS

(c) The effect of the ratio of axial moment of inertia of

(d) Similar to conventional spin-stabilized projectiles, dual-spin-stabilized projectiles also rely on extremely high spin rates to maintain gyroscopic stability. The resonance of dual-spin-stabilized projectiles may occur under some certain conditions, which could be complemented into present theoretical research for spin-stabilized projectiles.

PRESENTATION ENDS

Thank you for your attention !

Any questions are sincerely welcome !

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